A trading strategy with variable investment from minimizing risk to profit ratio

Stefan Liehr*, Klaus Pawelzik

Institute of Theoretical Physics, University of Bremen, 28334 Bremen, Germany

Received 30 April 2000; received in revised form 12 June 2000

Abstract

Assuming that financial markets behave similar to non-stationary random walk processes we derive an optimal trading strategy with variable investment for minimizing the risk to profit ratio over the trading period. We define a predictability measure which can be attributed to the deterministic and stochastic components of the price dynamics. The influence of predictability variations and especially of structures of short-term inefficiencies on the optimal amount of investment is analyzed in the given context. Finally, we show the performance of our trading strategy on an artificial price dynamics and on the DAX and S&P 500 as examples for real-world data using different types of prediction models in comparison. © 2000 Elsevier Science B.V. All rights reserved.

Keywords: Non-stationary random walk; Optimal trading strategy; Variable investment; Sharpe ratio; Prediction; Neural networks

1. Introduction

The theory of efficient markets denies the possibility for finding a profitable trading system which is able to forecast future market behavior [1]. In contrast to that theory there are successes of expert traders and also of trading systems which can only be explained by temporary inefficiency margins in the market and therefore by information not incorporated in the current market prices [2]. If such periods of inefficiency and the associated fluctuations of predictability are not recognized by the trading models they cause undesired variations in their performance. Besides maximization of profit, a very important objective for optimizing trading systems with finite reserve fund is therefore also a simultaneous minimization of risk.

* Corresponding author.
Since decades much work was done concerning the analysis of stock price fluctuations [3–8]. In recent years, significant deviations from the simple random walk assumption, e.g. Levy-like distributions of short-term price movements [9,10] and non-stationarity effects were found [11].

Inspired by these developments we use the concept of a non-stationary random walk with drift for the description of profit evolution. Non-stationarity is incorporated by time-dependent strengths of the stochastic and deterministic components of the random walk process. Both components can be identified with expected values of volatility and drift. Assuming a Gaussian distribution of returns we define a predictability measure based on the probability for gaining or losing money. As stated above we are looking for an objective function for finding the optimal amount of investment depending on the current market predictability such that the profit is maximized while the risk is minimized. We show that this objective is equivalent to the minimization of the time period of bankruptcy risk which is given by the risk to profit ratio (similar to the inverse Sharpe ratio $^1$ – a commonly used measure in finance). Finally, we present methods for training trading systems which include appropriate prediction models for an estimation of the characteristic features of the price dynamics. In context of the results on different time series, we emphasize the necessity of using trading models with variable investments in order to react to changing market predictability and to reduce the variance in gained profit.

The paper is organized as follows: In Section 2 we derive an optimal trading strategy with variable investment based on a non-stationary random walk assumption, in Section 3 we show different types of predictive models, in Section 4 we show results of simulations on artificially generated price evolutions and on the DAX and S&P 500 time series as two examples of real-world data. Section 5 summarizes our findings and Section 6 gives an outlook for further investigations.

2. Theory

We assume a trading system is given which provides an investment signal $a_t$ at each time step $t$ such that the resulting profits with respect to the returns are calculated by $g_t = a_t r_t$, where $r_t$ is the time series of returns $\{r_t\}_{t=1}^{T}$, with $r_t = \log(p_{t+1}/p_t)$ and prices $p_t$. The trading system can be any type of model which estimates an optimal signal $a_t = f(\Theta, x_t)$, $f : \mathbb{R}^m \times \mathbb{R}^n \rightarrow \mathbb{R}$, given an $n$-dimensional input vector $x_t$ and an $m$-dimensional parameter vector $\Theta$. The cumulated profit is then given by $G_t = \sum_{t=1}^{T} g_t$.

In order to find an optimization strategy for the trading system we are looking for an objective function which combines the maximization of profit with a minimization of risk during the trading process. Trading systems with constant investment signal only maximize profit without regard to risk variations. Therefore, systems with variable

$^1$Sharpe ratio $= (r(x) - r_0)/\sigma(x)$, with $r(x)$ the average rate of return of some investment $x$, $r_0$ the best available rate of return of a “risk-less” security like cash and $\sigma(x)$ the volatility (standard deviation) of the rate of return of investment $x$ [12].
investment have to be chosen, but the question is how to determine the amount of investment depending on actual predictability of the underlying processes.

The predictability will be defined as some function of the strength of stochastic and deterministic components which constitute two characteristic parts of profit evolution. The stochastic component incorporates all non-predictable fluctuations caused e.g. by new information entering the market or by irrationality in the investors' decision processes. The deterministic component is due to inefficiencies in the financial market which are detected by the trading system. The latter causes a positive drift in the cumulated profit. The relative strengths of both components, in general, exhibit a very complex time-varying behavior.

Much work demonstrated a close relation between diffusion processes in physical systems and the evolution of prices in financial markets [4]. Along these lines we assume that the evolution of profit can also be described by a stochastic process like a discrete non-stationary random walk given by

\[ g_t = \bar{\mu}_t \Delta t + \bar{\sigma}_t \Delta X_t \]

with constant time interval \( \Delta t \) between successive trading events and time-dependent expected profit \( \bar{\mu}_t \) and associated expected variance \( \bar{\sigma}_t^2 \). This type of non-stationarity is induced by state-dependent, and due to the dynamics also time-dependent variations of the random walk process. Because the investment signal \( a_t \) results from a functional dependency on the dynamical state \( x_t \) at time \( t \), the expected values in Eq. (1) can be attributed to estimates resulting from the distribution of returns at state \( x_t \) by \( \bar{\mu}_t = a_t \bar{r}_t \) and \( \bar{\sigma}_t^2 = a_t^2 \bar{\sigma}^2 _{r,t} \), where \( \bar{r}_t \) is the expected return and \( \bar{\sigma}^2 _{r,t} \) its variance (or squared volatility). The term ‘expected’ is used to emphasize that these values have to be estimated by the trading system. We assume a Wiener process for the random variable \( \Delta X_t \):

\[ \Delta X_t = \Psi \sqrt{\Delta t} \]

with

\[ \text{Prob}(\Psi \in [\phi, \phi + d\phi]) = \frac{e^{-\phi^2/2}}{\sqrt{2\pi}} d\phi = \Phi(\phi) d\phi. \]

In this framework the considered stochastic and deterministic components can be identified with the terms in Eq. (1). The following definition of predictability \( \hat{\omega}_t \) is based on the probability (or hit quota) \( \hat{\eta}_t \) of obtaining a positive profit at time \( t \):

\[ \hat{\eta}_t = \text{Prob}(g_t > 0) = \int_{-\infty}^{\bar{\mu}_t \sqrt{\Delta t} / \bar{\sigma}_t} \phi(\phi) d\phi = \frac{1}{2} + \Phi_0 \left( \frac{\bar{\mu}_t}{\bar{\sigma}_t} \sqrt{\Delta t} \right) \]

with \( \Phi_0(x) = \int^{x}_{-\infty} \phi(\phi) d\phi = \text{erf}(x) - \frac{1}{2} \). Predictability is then defined as

\[ \hat{\omega}_t = 2\hat{\eta}_t - 1 = 2\Phi_0 \left( \frac{\bar{\mu}_t}{\bar{\sigma}_t} \sqrt{\Delta t} \right). \]

This definition limits the predictability measure to \( \hat{\omega}_t \in [-1;1] \) whereby negative values can only be due to systematically wrong predictions of the trading system, which, by inversion, would also exhibit predictive power.
Fig. 1. Different realizations of a drifting random walk. Mean evolution ($\lambda = 0$) and confidence curves for $\lambda = \pm 2$ are additionally plotted.

Now we consider a confidence region for the stationary case of Eq. (1) with constant average profit $\mu_t = \hat{\mu}$ and constant variance $\sigma^2_t = \hat{\sigma}^2$. With

$$C_t(\lambda) = \hat{\mu}t + \lambda \hat{\sigma} \sqrt{t}$$

this region is given by the interval $G_t \in [C_t(-\lambda); C_t(\lambda)]$ where the cumulated profit at time $t$ is expected to be found with probability $\int_{-\lambda}^{\lambda} \phi(\phi) \, d\phi$. At the beginning of trading, $G_t$ has a significant component of negative profit (loss of money) which signifies a certain time of risk $t_0$ where $C_t(-\lambda)$ is negative. This corresponds to a substantial probability to go bankrupt if no sufficient amount of reserve fund $G_f$ is available. This is illustrated in Fig. 1. Time of risk and reserve fund are given by

$$C_{t_0}(\lambda) = 0 \Rightarrow t_0 = \frac{\lambda^2 \hat{\sigma}^2}{\hat{\mu}^2}$$

(7)

$$\left. \frac{C_t(\lambda)}{\partial t} \right|_{t_0} = 0 \Rightarrow t_f = \frac{t_0}{4} \Rightarrow G_f = -C_{t_f}(-\lambda) = \frac{\hat{\mu} t_0}{4}$$

(8)

In the stationary case there is no possibility for minimizing this time of risk because of the lack of variations in predictability. Transferring time of risk to the non-stationary random walk process over a time period $1 \leq t \leq T$ we have

$$t_0^{NS} = \frac{\hat{S}_T^2}{G_T} : V_T$$

(9)

with the mean expected profit $\hat{\mu}_t = 1/t \sum_{\epsilon=1}^{t} \hat{\mu}_\epsilon$ and mean expected variance $\hat{S}_t^2 = (1/t) \sum_{\epsilon=1}^{t} \hat{\sigma}_\epsilon^2$. Minimization of $t_0^{NS}$ means minimization of risk to profit ratio over the specified trading period which fulfills exactly the requirements for an optimal investment signal stated above. Therefore, we define $t_0^{NS}$ as the objective function $V_T$ for
the optimization process of the trading system. The optimal investment signals $a_t^*$ can be derived by the condition $\frac{\partial V_T}{\partial a_t^*} = 0$ which leads to

$$a_t^* = \frac{\dot{r}_t}{\sigma_t^2} \frac{V_T \hat{G}_T}{\sqrt{\Delta t}}. \quad (10)$$

The same proportionality $a_t^* \sim \dot{r}_t/\sigma_t^2$ fulfills the minimization condition for the reserve fund $G_f$. The expected optimal value of the objective function is then only given by the expected values of returns and their variances:

$$\breve{V}_T^* = \left( \frac{1}{T} \sum_{t=1}^{T} \frac{\dot{r}_t^2}{\sigma_t^2} \right)^{-1}. \quad (11)$$

The estimations of $\dot{r}_t$ and $\sigma_t$ have to be done by usual prediction systems which is subject of the next section.

The remaining question is how to evaluate the performance of the system. The optimal value $\breve{V}_T^*$ does not give any idea about its reliability. Therefore, we propose to use the realized risk to profit ratios as performance measures for the trading strategy with variable investment ($\breve{V}_T^*$) and its counterpart with constant investment ($\breve{V}_T^c$):

$$\breve{V}_T^* = \frac{(1/T) \sum_{t=1}^{T} a_t^2 (r_t - \dot{r}_t)^2}{((1/T) \sum_{t=1}^{T} a_t r_t)^2} \quad \text{and} \quad \breve{V}_T^c = \frac{(1/T) \sum_{t=1}^{T} (r_t - \dot{r}_t)^2}{((1/T) \sum_{t=1}^{T} \text{sgn}(a_t) r_t)^2}. \quad (12)$$

The ratio between the mean profits $\hat{G}_T^e$ and $\hat{G}_T^c$ gained by the strategies with variable and with constant investment signals gives another measurement for the success of our algorithm. We call this ratio the excess profit:

$$\delta G = \frac{\hat{G}_T^e}{\hat{G}_T^c} = \frac{\sum_{t=1}^{T} a_t r_t}{\sum_{t=1}^{T} \text{sgn}(a_t) r_t}. \quad (13)$$

### 3. Prediction models

For estimation of the expected return and volatility we compare two types of models. We construct the series of input vectors $x_t$ by embedding [13] the time series of returns into a space of embedding dimension $n_e$: $x_t = \{r_{t-1}; \ldots; r_{t-n_e}\} \in \mathbb{R}^{n_e}$. The target values are $y_t = r_t$ for the return prediction model and $z_t = (r_t - \breve{y}_t)^2$ for the volatility prediction model. The tilde denotes the actual prediction of the respective system.

#### 3.1. Discrete state space model

The first prediction model is a discrete state space model (DSS). Discretization is done by taking only the signs of each input component which leads to $2^{n_e}$ different states $s_t$. This transformation can be described by the mapping function $q: \mathbb{R}^{n_e} \rightarrow [1; \ldots; 2^{n_e}]$ with $s_t = q(x_t)$. The estimations of the model are given by the averages over all returns and variances associated to each state:

$$\hat{r}_t; \quad \breve{y}_t = \langle y_t \rangle_{q(x_t)}, \quad (14)$$

where $\langle \cdot \rangle_{q(x_t)}$ denotes the average over all returns and variances associated to each state $s_t = q(x_t)$. The estimations are then taken as additional input and can be considered as state dependencies in the model.
^2_{t'}: \tilde{z}_{t'} = \langle z_{t'} \rangle_{q(x_t)} \tag{15}

with \( \langle \cdot \rangle_{q(x_t)} \) as the average over all time steps \( t' \) whose state \( s_{t'} \) is equal to state \( s_t = q(x_t) \).

### 3.2. RBF model

The second kind of models we consider are radial basis function (RBF) neural networks of the Moody Darken type [14] with a slight modification for the case of the volatility model. Because volatility has to be strictly non-negative we use squared weight coefficients. Both models are given by

\[
\hat{r}_t: \tilde{y}_t = \sum_i w_i \frac{g_i(x_t)}{\sum_j g_j(x_t)} \quad \text{with} \quad g_i(x_t) = e^{-\frac{(x_t - z_i)^2}{2\sigma^2}} \tag{16}
\]

\[
\hat{\sigma}^2_{t'}: \tilde{z}_{t'} = \sum_i v_i \frac{h_i(x_t)}{\sum_j h_j(x_t)} \quad \text{with} \quad h_i(x_t) = e^{-\frac{(x_t - c_i)^2}{2\sigma^2}}. \tag{17}
\]

Training is performed as usual by an unsupervised adaptation of centers and width of Gaussians \( g_i \) and \( h_i \) using K-means clustering [15], afterwards a gradient descent training adjusts the second layer weights. Over-fitting is avoided by using only a small number of Gaussians and controlling the generalization performance on test data. The training process consists of two steps. First step is the training of the return model, second is training the volatility model on the errors of the return model.

### 4. Applications

In this section we demonstrate the performance of our trading strategy with an artificial data set and two real financial time series. Here, we compare our trading strategy with variable investment to a strategy with constant investment. The latter gives a constant investment signal in the direction of expected return. Therefore, it is based only on the sign of the return prediction models described before, and does not use any volatility information. For reason of comparability the investment signals of both strategies are normalized to the same amount of mean absolute investment over the whole trading period. In the case of variable investment, the maximal amount of absolute investment is limited to five times its mean because also in real trading the possible investment is in general restricted to some maximum.

For testing the generalization ability we use the first 70\% of the data in the training procedure, the remaining patterns are excluded from training and used for the test purpose. The time series of returns are transformed to zero mean, so that all profit gained by the trading system is excess profit exceeding the possible profit by the simple usage of a buy-and-hold strategy.

\[^2_x := x \cdot x \text{ (scalar product).}\]
4.1. Artificial data

In order to show the suitability of our trading strategy to price dynamics with varying predictability, we apply it to an artificial time series which obeys the characteristics of a non-stationary random walk like the one described in Section 2:

\[ r_t = \tilde{r}_t \Delta t + \tilde{\sigma}_t \Delta X_t. \] (18)

Caused by the Wiener process, the returns \( r_t \) are Gaussian distributed with variance \( \tilde{\sigma}_t^2 \Delta t \) and mean \( \tilde{r}_t \). The time-varying behavior of \( \tilde{r}_t \) and \( \tilde{\sigma}_t \) is determined by a Markov model with discrete state space: Depending on the history of the last \( n_s \) signs of \( r_t \), a couple \( (\tilde{r}_t, \tilde{\sigma}_t) \) out of \( 2^{n_s} \) predefined possibilities and thereby a certain predictability according to Eq. (5) is chosen. This is illustrated in Fig. 2. The model is initialized by assignment of \( n_s \) normally distributed random numbers to \( r_1, \ldots, r_{n_s} \). The time series used for simulated trading is taken after a certain amount of iterations in order to avoid transient dynamics due to the initialization procedure.

Fig. 3a shows a concrete simulation of that process over 2000 time steps with \( n_s = 3, \Delta t = 1/250 \), predictabilities \( \omega_t \) randomly chosen in the interval \([-0.2; 0.2]\), volatilities randomly chosen in \([0.1; 0.3]\) and \( \tilde{r}_t \) is given by Eq. (5). The embedding procedure uses the same history as the data generation process \( n_e = n_s = 3 \), the RBF network has 10 hidden units, hence RBF and DSS models have a comparable amount of adaptation parameters. The small excess of RBF parameters are chosen in order to achieve a tolerable coverage of state space.

The results of trading on the artificial data set are shown in Fig. 3 and Table 1. The discrete state space model with variable investment exhibits systematically lower risk to profit ratios than the RBF network. This could be easily understood because the DSS model is the generic model of that data set. Further, both models clearly show a better performance with the variable investment strategy compared to constant investment. In all cases variable investment leads to higher mean profits than a constant strategy \((\delta G > 1)\). The exact agreement between \( \hat{V}_T^\gamma \) and \( V_T^\gamma \) of the DSS model on the
Fig. 3. (a) Evolution of cumulated zero mean return (larger frame) and return (smaller frame) of the artificial data set (2000 steps of a non-stationary random walk based on a Markov model with $2^3$ different states defined by the sign history). (b),(c) The absolute investment signal and cumulated profit of trading on the artificial data set with a discrete state space model (b) and an RBF neural network with 10 hidden units (c). Training data are the first 70%, the remaining data are used for the test purpose. Evolution of expected profit and the expected variance (2σ-confidence region) curves are plotted additionally during the test period.
Table 1
The quantitative results of risk to profit ratios and excess profits are shown for all simulations

<table>
<thead>
<tr>
<th>Artifical</th>
<th>DAX</th>
<th>S&amp;P 500</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Train</td>
<td>Test</td>
</tr>
<tr>
<td>DSS</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\hat{V}_T^*$</td>
<td>22.5</td>
<td>22.5</td>
</tr>
<tr>
<td>$V_T^*$</td>
<td>22.5</td>
<td>52.2</td>
</tr>
<tr>
<td>$V_T^c$</td>
<td>36.8</td>
<td>290.7</td>
</tr>
<tr>
<td>$\delta G$</td>
<td>1.36</td>
<td>2.59</td>
</tr>
<tr>
<td>RBF</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\hat{V}_T^*$</td>
<td>51.8</td>
<td>53.9</td>
</tr>
<tr>
<td>$V_T^*$</td>
<td>46.2</td>
<td>123.8</td>
</tr>
<tr>
<td>$V_T^c$</td>
<td>86.0</td>
<td>235.3</td>
</tr>
<tr>
<td>$\delta G$</td>
<td>1.45</td>
<td>1.50</td>
</tr>
</tbody>
</table>

training data is systematic through all simulations. This can be explained directly by the mathematical correspondence between the structures of the DSS model and the data generation process. All other types of models show values $V_T^c$ which differ from the expected optimum because of systematic bias and a finite number of data. Whether $V_T^c$ is higher or lower than $\hat{V}_T^*$ depends on the balance of an over- and underestimation of returns and volatilities associated with the distribution of price returns.

Fig. 3 shows the profit evolution of both models over the whole time series. Additionally, the expected evolution and the $2\sigma$-confidence curves defined in Section 2 are plotted for the last 30% (test data). At each time step these confidence curves limit the interval, where 95% of all profit evolutions starting at the beginning of the test period are located in.

In a preliminary conclusion we could demonstrate with this data set that our trading strategy with variable investment and an appropriated prediction model is capable of finding the relevant dynamical structures of a time series and that it minimizes the risk to profit ratio.

4.2. DAX

The available time series of German Stock Market Index (DAX) consists of 2340 daily close prices of an almost 10 year period from 11/26/1990 until 03/17/2000. The time series of zero mean returns is shown in Fig. 4a. We use an embedding dimension of $n_e = 5$ for adaptation of the different types of prediction models. In order to obtain comparability between DSS model and RBF network we choose 32 hidden units for the RBF structure which equalizes exactly the respective amount of adaptation parameters.

The results on the DAX are not as systematic as on the artificial data set which can be seen in Fig. 4 and Table 1. First of all, on the training data both types of models clearly show a better performance when using the variable strategy. Further, the risk to profit ratios using the DSS model are better than those of the RBF network, which shows that looking only on the sign history is not as bad as one might expect, indeed it is surprisingly good.
Fig. 4. (a) Evolution of cumulated zero mean return (larger frame) and return (smaller frame) of the DAX (2340 close prices from 11/26/1990 until 03/17/2000). (b),(c) The absolute investment signal and cumulated profit of trading on the returns of DAX close prices with a discrete state space model (b) and an RBF neural network with 32 hidden units (c). State space is reconstructed by embedding with dimension n_e = 5. Training data are the first 70%, the remaining data are used for the test purpose. Evolution of expected profit and the expected variance (2σ-confidence region) curves are plotted additionally during the test period.
However, on the test data the RBF network outperforms the DSS model clearly which can also be seen in Fig. 4b and c. The DSS model shows much stronger fluctuations and leaves the confidence region two times with high significance. This can be explained by the occurrence of the large crash in August 1998, located in the test data just around time step 1900. Parts of the downward dynamics of this crash are predicted quite well by both models with high investments yielding a positive jump in profit evolution. Despite this correct prediction, both models exhibit deviations between expected and realized profit evolutions before and after the crash. The DSS model loses money and leaves the confidence region very fast. The consequence is a huge risk to profit ratio and an excess profit $\delta G$ near unity. The RBF network behaves more moderate and just gains zero profit during that time period.

The inclusion of the crash region into the training data by using the first 30% of the data for the test purpose and the remaining data for training leads to a strong reduction of the risk to profit ratio in the case of the DSS model. The $V_T^\tau$ value changes substantially from 2009.4 to 333.9 for the new test data set while on the training data set the value increases only to 54.0.

The behavior of the models indicates that the crash is accompanied by new dynamical features unknown to the models. Such types of non-stationary processes, which are not included in the training data are always a problem in modeling economical data. With our trading strategy we can deal even with such situations by using the confidence region for evaluation of the trading system at each time. If the realized profit evolution leaves that confidence region it can be used as a signal for the occurrence of dynamical processes not known from the training data and for possibly triggering a new training procedure.

Besides these difficulties in the prediction of the test data, the RBF network gains a considerable absolute profit in contrast to the DSS model. The main contributions come from the period just at the beginning of the test data, the final part of the crash and a recognized dynamical structure quite at the end of the time series.

### 4.3. S&P 500

The available time series of the Standard & Poor’s 500 Index $^3$ (S&P 500) consists of 5109 daily close prices of about a 20 year period from 02/01/1980 until 03/17/2000. The time series of zero mean returns is shown in Fig. 5a. Again, we use an embedding dimension of $n_e = 5$ for adaptation of the different types of prediction models and 32 RBF centers by the same reason as stated before.

Compared to the discussion of trading on the DAX, the results in Fig. 5 and Table 1 for trading on the S&P 500 data set are more clear and easier to interpret. Taking a look at the risk to profit ratios, both models with variable strategy are similar in their performance, usage of the constant strategy leads to a significant deterioration. Excess profit is higher for the RBF network than for the DSS model.

---

$^3$ A daily measure of share market performance, based on the performance of 500 major companies.
Fig. 5. (a) Evolution of cumulated zero mean return (larger frame) and return (smaller frame) of the S&P 500 (5109 close prices from 02/01/1980 until 03/17/2000). (b),(c) The absolute investment signal and cumulated profit of trading on the S&P 500 with a discrete state space model (b) and an RBF neural network with 32 hidden units (c). State space is reconstructed by embedding with dimension $n_e = 5$. Training data are the first 70%, the remaining data are used for the test purpose. Evolution of expected profit and the expected variance (2σ-confidence region) curves are plotted additionally during the test period.
The superiority of the RBF network becomes more clear if one considers the cumulated profit evolution together with its absolute value at the end of the test period. The curve of realized profit does not leave the confidence region which indicates a quite good generalization ability of both models. Further, the RBF network can take advantage of certain dynamical features of increased predictability in the middle of the test period. Especially, the latter contributes to a higher profit of the RBF network in comparison to the DSS model.

The crash during the training period at time step 2000 (October 1987) is captured well by both models but with higher extent by the RBF network than by the DSS model. This can be explained by the discrete structure of the DSS model looking only on the signs of price returns. Slight downward tendencies with successive negative returns cannot be distinguished from huge crashes because both exhibit the same sign history. This results in a moderate investment for both dynamical processes while the RBF network is able to separate both cases and can react with adequate investments.

Motivated by the discussion of the crash in the DAX time series and its influence depending on the segmentation of the data set, we trained another trading system with the S&P 500 crash excluded from the training procedure. The 30% test data are located symmetrically around data point 2000, the remaining data are used for training. On the new test data set, the $V_T$ values increase strongly to 425.9 (DSS) and 4268.7 (RBF), while they change slightly to 60.0 (DSS) and 94.0 (RBF) on the training data set. This result confirms our expectations and the well-known fact that crashes are accompanied by changes in dynamics.

5. Summary

We derived an optimal strategy for trading in financial markets which are subject to changes in predictability of their dynamics. Based on the assumption of non-stationary stochastic processes we optimized a variable investment system with the objective to minimize the risk to profit ratio. We showed that the objective can also be understood in terms of a minimization of the risk to go bankrupt or equivalently a minimization of the necessary reserve fund.

The presented framework can be used for an estimation of the confidence level of a trading system by using the variance curves given by the random walk assumption. Whenever the profit evolution crosses the border of the confidence region that can be interpreted as a signal for re-training the prediction models.

We demonstrated the performance of our trading strategy on an artificial data set as well as on two real financial time series – the DAX and the S&P 500. We could show that the variable investment strategy and the used prediction models could mostly capture the main dynamical characteristics of the time series. We want to emphasize that the results of our simulations are taken without great scanning effort on the parameter regime, like embedding dimension and number of RBF hidden units. And we suppose
that further improvements of reliability and general performance can be reached by more work in that area and an additional usage of higher frequency data for training the prediction models.

6. Outlook

The prediction models used in this work can easily be replaced by other, more sophisticated models. Especially, mixture models for adaptation to fundamental switching processes between different dynamical regimes could be considered [16,17]. In our future work we plan to develop a direct optimization algorithm for the investment signal, thereby avoiding the separate estimation of the market behavior by two prediction models, one for the expected return and one for the volatility. Finally, the investigation of other volatility distributions like the Levy distribution which contains the Gaussian normal distribution as a special case could yield a more precise description of the market dynamics especially on shorter time scales [9,18].

Acknowledgements

We acknowledge support of the Deutsche Forschungsgemeinschaft (DFG, grant Pa569/2-1).

References

