Dynamics of competition between collectivity and noise in the stock market

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Abstract

Detailed study of the financial empirical correlation matrix of the 30 companies which Deutsche Aktienindex (DAX) comprised during the period of the last 11 years, using the time window of 30 trading days, is presented. This allows clear identification of a nontrivial time-dependence of the resulting correlations. In addition, as a rule, the drawdowns are always accompanied by a sizable separation of one strong collective eigenstate of the correlation matrix which, at the same time, reduces the variance of the noise states. The opposite applies to drawups. In this case, the dynamics spreads more uniformly over the eigenstates which results in an increase of the total information entropy. Analogous study of the market corresponding to Dow Jones industrial average (DJIA) leads to similar conclusions. In the latter case, however, the correlations are weaker on average. One possible reason for this effect is that the market represented by DJIA is less susceptible to various external factors than the one represented by DAX. © 2000 Elsevier Science B.V. All rights reserved.

1. Introduction

Studying correlations among various financial assets is of great interest for both practical and fundamental reasons. Practical aspects relate, for instance, to the theory of optimal portfolios [1,2] and risk management. The fundamental interest, on the other hand, results from the fact that such study may shed more light on the universal aspects of evolution of complex systems. Recent studies [3,4] of the related problems in the context of the stock market show that majority of eigenvalues in the spectrum of the correlation matrix agree very well with the universal predictions of random matrix...
theory [5]. Locations of some of the eigenvalues differ however from these predictions and thus identify certain system-specific, nonrandom properties of the system. The corresponding eigenvalues thus carry most of the information about the system. The above studies however have a global in time character and do not account for a possible change of correlations on shorter time scales.

For a price \( x_i(t) \) of the \( i \)th asset at time \( t \), one defines its return \( G_i(t) \) as

\[
G_i(t) = \ln x_i(t + \tau) - \ln x_i(t) \approx \frac{x_i(t + \tau) - x_i(t)}{x_i(t)},
\]

where \( \tau \) is also called the time lag. The normalized returns, with the average value subtracted and its variance normalized to unity, are defined by

\[
g_i(t) = \frac{G_i(t) - \langle G_i(t) \rangle_t}{\sigma(G_i)}, \quad \sigma(G_i) = \sqrt{\langle G_i^2(t) \rangle_t - \langle G_i(t) \rangle_t^2},
\]

where \( \sigma \) is volatility of \( G_i(t) \) and \( \langle \ldots \rangle_t \) denotes averaging over time. Now, one can form \( N \) time series \( [g_i(t_0), g_i(t_0 + \tau), \ldots, g_i(t_0 + (T - 1)\tau)] \) of length \( T \), i.e., an \( N \times T \) rectangular matrix \( M \). Then, the correlation matrix is defined as

\[
C = \frac{1}{T}MM^T.
\]

This is equivalent to the standard definition of the correlation matrix

\[
C_{ij} = \frac{\langle G_i(t)G_j(t) \rangle_t - \langle G_i(t) \rangle_t\langle G_j(t) \rangle_t}{\sigma(G_i)\sigma(G_j)}.
\]

In order not to artificially reduce the rank of this matrix, \( T \) needs to be at least equal to \( N \). This sets the lowest limit on a time window which can be used to study the time-dependence of correlations. One of the best examples where \( T \) can be set relatively low, and thus efficiently allow to get some time-dependent picture of correlations versus the global market index, is provided by Deutsche Aktienindex (DAX). It represents a matured, relatively independent market, whose behavior is well reflected by \( N = 30 \) companies defining this index. During the period studied here it displays the whole variety of behaviors like stagnancy, booms, including a pattern of self-similar log-periodic structures [6], and crashes.

2. Empirical facts

2.1. Correlation matrix

The present study is based on daily variation of all \( N = 30 \) assets of the DAX during the years 1988–1999. The time interval (time window) \( T \) is set to 30 and continuously moved over the whole period. That the character of correlations may significantly vary in time is indicated in Fig. 1 which displays some typical distributions of matrix elements of \( C \) in three different cases: (i) an average over all time windows of length \( T = 30 \), (ii) for a single \( T = 30 \) time window which ends on November 25, 1997,
Fig. 1. Distributions of matrix elements of the correlation matrix $C$ calculated from the daily price variation of all $N=30$ companies comprised by DAX in three different cases: (i) an average over all time windows of length $T=30$ during the period 1988–1999, (ii) for a single $T=30$ time window which ends on November 25, 1997, (iii) for a single $T=30$ time window which ends on April 7, 1998. Clearly, in all the cases the distributions are Gaussian-like but their variance and location is significantly different. In fact, a distribution of this type prescribes the structure of the corresponding eigenspectrum. The point is that to a first approximation any of such matrices can be represented as

$$C = G + \gamma U,$$

where $G$ is a Gaussian centered at zero and $U$ is a matrix whose all entries are equal to unity and $0 \leq \gamma \leq 1$. The rank of the matrix $U$ is one, therefore the second term alone in the above equation generates only one nonzero eigenvalue of magnitude $\gamma$. As the expansion coefficients of this particular state are all equal this assigns a maximum of collectivity to such a state. If $\gamma$ is significantly larger than zero the structure of the matrix $C$ is dominated by the second term in (5) and an anticipated result is one collective state with large eigenvalue. Since in this case $G$ can be considered as only a ‘noise’ correction to $\gamma U$, all the other states are connected with small eigenvalues. The above provides an alternative potential mechanism for emergence of collectivity out of randomness to the one taking place in finite interacting Fermi systems. Here, a reduction of dimensionality of a leading component in the Hamiltonian matrix is associated with appearance of more substantial tails in the distribution of large matrix elements as compared to an ensemble of random matrices [7].

2.2. Structure of eigenspectrum

The Gaussian-like shape of the distribution of matrix elements of $C$ also makes clear that the bulk of the spectrum must be consistent with predictions of the Gaussian orthogonal ensemble (GOE) [8] of random matrices. Independently, this fact
is already well established in the recent literature for the financial market [3,4] and seems quite generic in the context of biological systems [9]. In the present case instead, we concentrate on tracing a possible nonstationarity in the location of eigenvalues. The corresponding central result of our paper is displayed in Fig. 2. As it is clearly seen, the drawups and the drawdowns of the global index (DAX), respectively, are governed by dynamics of significantly distinct nature. The drawdowns are always dominated by one strongly collective eigenstate with large eigenvalue. Such a state thus exhausts a dominant fraction of the total portfolio variance

$$\sigma_P = \sum_{i,j} p_i C_{ij} p_j,$$

where $p_i$ expresses a relative amount of capital invested in the asset $i$, and $C_{ij}$ are the entries of the correlation matrix $C$ [1,10] (this becomes obvious by transforming $C$ to its eigenbasis). The more dramatic the fall is, the more pronounced is this effect. At the same time, by conservation of the trace of $C$, the remaining eigenvalues (representing some less risky portfolios) are compressed in the region close to zero. In a formal sense, this effect is reminiscent of the slaving principle of synergetics [11,12]: one state takes the entire collectivity by enslaving all the others.
The opposite applies to drawups. Their onset is always accompanied by a sizable restructuring of eigenvalues towards a more uniform distribution. The related principal effect is that the largest eigenvalue moves down which is compensated by a simultaneous elevation of lower eigenvalues. At one instant of time (mid-1996), which marks the beginning of the long-term boom, the two largest eigenvalues became almost degenerate. Based on these results a general statement that an increase in the market involves more competition than a decrease, seems appropriate since in the former case the total variance is more democratically distributed among eigenstates of the correlation matrix. In other words, the increase in the stock market never involves parallel uniform increase of prices of all the participating companies as it happens during decreases.

Such a conclusion is also indicated by the information entropy

$$I_k = - \sum_{l=1}^{30} (u_{kl})^2 \ln(u_{kl})^2,$$

where $u_{kl}$ (here $l = 1, \ldots, 30$) are the components of the $k$th eigenvector. Its GOE limit [13] is

$$I_{\text{GOE}}^{\text{GOE}} = \psi(N/2 + 1) - \psi(3/2),$$

where $\psi$ is the digamma function and $N$, in the present study, corresponds to the number of stocks. For $N = 30$ we thus have $I_{k}^{\text{GOE}} \approx 2.67$ and for the uniformly distributed components ($u_{kl}^2 = 1/30$) $I_k^{\text{unif}} = 3.4$. These two limits are to be related to the information entropies displayed in the lower panel of Fig. 2. As one can see, it happens only during decreases that the information entropy approaches the limit of uniform distribution for the upper, most collective state. Otherwise this state becomes somewhat more localized. The information entropy of the 'noise' states on the average agrees with $I_{\text{GOE}}^{\text{GOE}}$, though, a more careful inspection shows systematic and consistent deviations, going in opposite directions during increases and decreases, respectively.

In quantitative terms this effect can be easily deduced by looking at the total information entropy

$$I_{\text{tot}} = \sum_{k=1}^{30} I_k$$

shown in Fig. 3. On average it always moves in opposite direction relative to the information entropy $I_1$ of the most collective state, even though $I_1$ is included in $I_{\text{tot}}$. This result is very interesting and even intriguing in itself. The market drawups are accompanied by increase of the total information entropy while drawdowns are associated with its decrease. At first glance such a behavior and, especially, the entropy decreases accompanying such turmoils as crashes may look somewhat embarrassing. The truth however is that prices and related quantities reflect only a part of the market world. There is also an environment with which any market constantly interacts and which easily may absorb a corresponding portion of entropy. Actually, the turmoils accompanying crashes are visible more in the market environment than in the genuine market parameters.
Since the structure of the correlation matrix is influenced by measurement noise more for short time series than for the long ones, a question which needs to be answered is to what extent our conclusions are stable with respect to the length $T$ of the price time series. Of course, the specific values of the quantities considered do depend on $T$ but the main effect of increasing it is to smear them out in time. The global tendencies of interest for the present paper, remain however unchanged. An example is shown in Fig. 4 which displays the structure of eigenspectrum of the correlation matrix for several values of $T$. 
2.3. DAX versus DJIA

Another independent market which comprises same number of 30 companies, and thus allows a similar time resolution of correlations, corresponds to the Dow Jones industrial average (DJIA). The time-dependence over the past decade of eigenspectrum of its correlation matrix is shown in Fig. 5 where, as before, the time window of $T=30$ trading days is used. Concerning the mechanism governing evolution of increases and decreases, respectively, the same tendency as for the DAX is clearly seen here as well. There is however one interesting difference. On average the correlations are significantly weaker for the DJIA. This is expressed by smaller magnitude of the average separation between the largest eigenvalue and the remaining ones. Also the two largest eigenvalues approach each other more often for DJIA than for DAX. This turns out to be consistent with the structure of the power spectra calculated from the corresponding minute time-variation of the global indices under consideration. These power spectra are displayed in Fig. 6. For reference the Standard and Poors 500 (S&P500) power spectrum is also shown. While all these power spectra are of the $1/\omega^2$-type, they reveal a significantly different dispersion which can be ascribed to a different noise content. The broadest dispersion in case of S&P500 seems natural since this index involves a much larger number of the companies. The other two, DAX and DJIA, include however the same number of components but dispersion of the DAX power spectrum is visibly narrower. One possible explanation for this effect is that the market represented by DAX is more influenced by external news which makes its components to evolve more coherently. This narrowing of the DAX power spectrum relative to DJIA is reminiscent of a biased Brownian motion, by an external field for instance.
2.4. Short time correlations

The above study of correlations among the financial assets is based on their daily price variation ($\tau = 1$ trading day). A natural related question is how do the relevant characteristics change when the price returns are taken over some shorter time lags $\tau$. It turns out that the correlations generically degrade with decreasing $\tau$. The distribution of entries of the correlation matrix evaluated from minute returns ($\tau = 1$ min) is Gaussian shaped and its location is systematically much more symmetric relative to zero than in the case of daily price returns (Fig. 1). Consistently, the largest eigenvalue typically assumes significantly lower values. An example of how this principal measure of collectivity behaves for various values of $\tau$ is illustrated in Fig. 7. Clearly, with increasing distance between the time points used to determine the magnitude of returns the correlations get amplified while the volatility, $\sigma(\dot{x}_1)$, gets smaller. This indicates that the collective effects are driven mainly by the longer term trends rather than by the short-term fluctuations.

3. Summary

The present study discloses several interesting novel facts about the dynamics of financial evolution. These empirical facts, interpreted in terms of the co-existence of collectivity and noise in correlations among the financial assets, provide arguments for distinct nature of the mechanism governing financial increases and decreases,
Fig. 7. Time dependence of the maximal eigenvalue of the DAX correlation matrix for $\tau = 1$ min (A), $\tau = 10$ min (B), $\tau = 30$ min (C) and $\tau = 60$ min (D) during the 50 h period at the end of September 1999. $T = 30$ is consistently used in all these cases. The dotted lines display the average (aver.) value and the volatility (volat.) is given for all cases.

respectively, even though such correlations on average are largely compatible with the random matrix theory predictions. The structure of eigenspectrum of the correlation matrix and the information entropy arguments point to increases as those phenomena which internally involve more diversity and competition as compared to decreases. It seems likely that such characteristics may apply to the dynamics of evolution of other complex systems as well.
References